Stackelberg Game Based Resource Pricing and Scheduling in Edge-Assisted Blockchain Networks

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***Abstract*—Currently, the blockchain, as a key enabling technology of digital currency, has attracted lots of attention from both industry and academia. The blockchain mining process requires high computing power to solve a Proof-of-Work (PoW) puzzle, which is hard to implement on users' mobile devices. So these miners may leverage the resources of the edge/cloud service providers (ESPs/CSP) to calculate the PoW puzzle. The existing edge-assisted blockchain networks simply assumed that all ESPs have a uniform propagation delay, which is not realistic. In this paper, we consider a more practical scene where ESPs with distributed geographic locations have diverse propagation delays when supporting the computation of the PoW puzzle. Additionally, the blockchain mining process generally involves the complicated competition and game among these ESPs and miners. Each ESP focuses on how to determine his resource price and to select the requests from the miners, so that he can maximize his utility. According to the set resource price, each miner concentrates on scheduling his resource requests for each ESP to maximize his individual utility which depends on ESPs' resource price and propagation delays. We model such a resource pricing and scheduling problem as a multi-leader multi-follower Stackelberg game and aim at finding the joint maximization of the utilities of each ESP and each individual miner. We prove the existence and uniqueness of the Stackelberg equilibrium (SE) and meanwhile propose an algorithm to achieve the corresponding SE. Finally, extensive simulations are conducted to verify the significant performance of the proposed solution.**

***Index Terms*—Blockchain, edge computing, game theory, resource pricing, resource scheduling, propagation delay.**

I. INTRODUCTION

In the past few years, Bitcoin has been widely used owing to its decentralized particularity. As one popular digital cryptocurrency, Bitcoin can be used across countries without worrying about being frozen by any financial institutions [1], and also can record and store all digital transactions in a decentralized append-only public ledger called "blockchain". Blockchain technology is applied in the Bitcoin field to record transactions and prevent tampering. Specifically, the data of digital transaction was packaged in the form of the linked blocks, in which each block is encrypted by using the Hash technique to ensure its security.

Due to the decentralized idea in the Bitcoin networks, it needs someone to collect the transaction records that occurred in the past period of time, package them into a block, and then link this block to the end of the existing blockchain. This is the most significant process of the blockchain network, which is called the blockchain mining process. In the mining process, miners are first required to solve a computationally challenging Proof-of-Work (PoW) puzzle. Then, each miner propagates his mined block to all blockchain network users to make this block be verified as soon as possible. This is because only when a block is verified by the majority of miners in this network, it can be considered to be added to the end of the blockchain successfully. In other words, the consensus protocol of blockchain can be realized. In fact, only the miner who successfully links a block to the exiting blockchain can gain a certain amount of Bitcoin in return as the mining incentive.

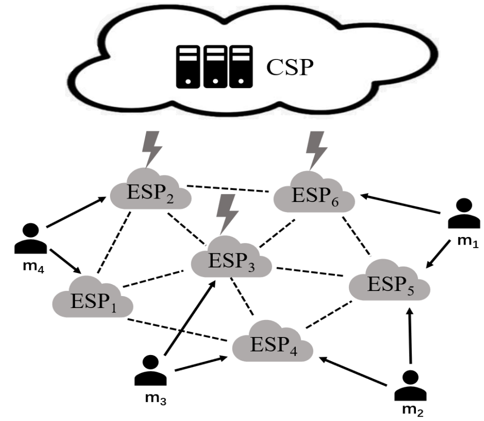

Fig. 1. Edge-Assisted Blockchain Mining Networks.

The blockchain based on PoW is a critical technology, which is considered as a technological innovation in the peer-to-peer network [2]. The security and reliability are thus ensured by this mechanism which requires numerous trial for a valid solution [3]. However, the blockchain based on PoW needs a mass of computation and storage resources. This is hard to be satisfied with a miner's terminal devices. Thanks to the development of edge/cloud computing technique, the miners can take on lease some on-demand resources from the edge/cloud service providers (ESPs/CSP) [4], so that they can efficiently complete the mining process. This is so-called edge-assisted blockchain mining networks, as shown in Fig.1.

The mining process in the edge-assisted blockchain networks is described as a speed game. First, miners send their computing requests to ESPs and purchasing some computation and storage resources on ESPs to calculate the PoW puzzle. Then, if any ESP calculated the PoW puzzle, he needs to propagate the block to all of the other ESPs in this edge-assisted blockchain network as soon as possible, so that he can make this block become the first one to realize the consensus principle. The miner who packages the block on this ESP and successfully takes the lead in reaching consensus principle is considered as the winner of the mining process. Agreeing on an identical blockchain by all nodes is also called "block convergence". Here, a new block will be validated earlier by other nodes if it can be spread to the whole blockchain network faster [5]. The block convergence of blockchain may be disrupted by the increased network latency (i.e., propagation delay). In other words, even if two nodes solve the PoW problem at the same time, the block packaged by one node may be discarded because the propagation time is longer than that of the other block.

The existing edge-assisted blockchain networks simply assumed that all ESPs have a uniform propagation delay, which is not practical in the real world. In this paper, we consider a price-based resource management mechanism with propagation delay in edge-assisted blockchain networks, in which ESPs have different propagation delays due to the different geographic locations in the edge computing networks. Further, we propose a multi-leader multi-follower Stackelberg game model between the computing service providers and miners.

The contributions of this paper are summarized as follows:

- We consider a three-layer edge-assisted blockchain mining network model, i.e., miners, ESPs and CSP. Each miner studies how to maximize his individual utility which depends on the resource price and the propagation delay of each ESP, while all ESPs focus on the computing resource pricing and scheduling to maximize their utility. We are the first to consider the impact on block convergence of different propagation delays due to the distributed geographic locations. Actually, ESPs' propagation delays will greatly affect the probability of blockchain mining success.
- To solve the competition and game among the ESPs and miners, we propose a special multi-leader multi-follower two-stage Stackelberg game model, in which the ESPs and miners are seen as the leaders and followers, respectively. The proposed model takes the ESPs' propagation delays and the impact of CSP into consideration when calculating the miners' utility.
- We derive the explicit-form expressions of the most beneficial price strategies for each ESP and at the same time the optimal resource requests for each individual miner. Furthermore, we analyze the existence and uniqueness of the Stackelberg equilibrium (SE), based on which we propose a corresponding algorithm to obtain the SE.
- Extensive simulations are conducted to verify the significant performance of the proposed solution.

II. System Model and Problem Formulation

A. Edge-Assisted Blockchain Mining Networks

In this paper, we consider the public blockchain mining networks based on PoW consensus protocol. In the blockchain mining networks, there are many ordinary network users, called miners, trying to complete the transaction package, called block, to purse some rewards. More specifically, the success of a miner appending the block to the end of the current blockchain contains two steps. 1) The miner needs to solve the PoW puzzle to ensure the security and validity, which is called mining procedure. 2) The miner must broadcast his results to the other network users in the blockchain network, which is called broadcasting procedure, so that the consensus principle can be realized. During the mining procedure, the PoW puzzle that the miners try to solve highly depends on the computation resources of the miners' terminal devices. In other words, the miners with more computation resources will have a larger probability of solving this PoW puzzle. However, the computation resources of a miner's terminal device are generally limited. The miners can take on lease some on-demand resources from ESPs, such that they can efficiently complete the mining process. Note that the ESPs are geographically distributed at network edge, so the network users can access the ESPs via the wireless local area networks. While these ESPs connect to the remote CSP through a core network, as shown in Fig. 1. In general, an ESP has a limited computation resource capability while the CSP is assumed to have unconstrained computation resources. For an ESP, when the total resources requested from the miners exceed his capacity, he will upload part of his requests to the CSP.

We consider there are n ESPs in the edge-assisted blockchain network, denoted as $\mathcal{N} = \{1, \cdots, j, \cdots, n\}$, and there are m miners in the blockchain system, denoted as $\mathcal{M} = \{1, \cdots, i, \cdots, m\}$. To complete the PoW puzzle, the miners will purchase computing service from ESPs or CSP. When the ESPs or CSP who calculates out the PoW puzzle, he will try to broadcast its result to all miners as soon as possible. In such a way, the corresponding miner who rents the computation resources may become the first one to realize the consensual block ahead of other competitors. In the blockchain networks, only the first miner who reaches the consensual block principle can obtain the reward. Note that in addition to the time of calculating out the PoW puzzle, the propagation delay of spreading the results to other miners is also an important factor. In the system model, we assume that the n ESPs are sorted in descending order of the propagation delay. Also, the propagation delay of the CSP is obviously greater than that of any ESP due to the remotest location.

The ESPs first set their unit price of selling their computing resources, and then the miners determine their request for each ESP according to the set price. Here, for each miner i, we use $X_i = (x_i^1, x_i^2, \cdots, x_i^n)$ to denote the request for these n ESPs, and use B_i to denote his budget. We suppose that each ESP in the edge-assisted blockchain mining networks has the same unit computing power but different from that of CSP. In fact, the probability of solving the PoW problem is related to computing power. In other words, the effective computing power of one ESP is proportional to the computation resources that the corresponding miner rents. As a result, the probability of miner i calculating the PoW problem on ESP j, denoted as α_i^j, indicates the percentage of the resources he rents on ESP

TABLE I
DESCRIPTION OF COMMONLY-USED NOTATIONS.

Variable	Description
$\mathcal{M}$, $\mathcal{N}$	The sets of miners and ESPs, respectively.
i, j	The indexes for miners and ESPs.
x_i^j	The service demand of miner i for ESP j.
X_i	The service demand of miner i for all ESP.
K_j	The maximum capacity of ESP j.
B_i	The budget of miner i.
t_j / p_j	The propagation delay / unit price of ESP j.
τ_i^j	Miner i' demand for ESP j is accepted or not.
c / h	The unit cost for providing service on ESP / the unit cost for uploading the request to CSP.
E_{all}	Total request accepted by all ESPs from all miners.
C_{all}	Total request uploaded to CSP from all ESP.
W_i^j	The probability of miner i winning on ESP j.
W_i	The probability of miner i
R	Reward of blockchain mining successfully.
U_i / V_j	The utility of miner i / ESP j

j to the total computation resources in ESPs, that is:

$$\alpha_i^j = \frac{x_i^j}{\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}}\tau_i^j x_i^j}. \tag{1}$$

Here, $\tau_i^j = 1$ means the request of miner i for ESP j is accepted by ESP j; on the contrary, $\tau_i^j = 0$ denotes this demand is uploaded to the CSP by ESP j. We use E_{all} to represent the total request accepted by all ESPs in this edge-assisted blockchain network, i.e., $E_{all} = \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}}\tau_i^j x_i^j$. Similarly, we use C_{all} to express the total request uploaded to the CSP by all ESPs due to their limited computing resource capacity, i.e., $C_{all} = \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{N}}\left(1-\tau_i^j\right)x_i^j$.

As introduced above, the process of a miner winning the reward consists of two procedures, i.e., the mining procedure and broadcasting procedure. The winning probability of broadcasting procedure is affected by the propagation delay. In fact, the long propagation delay may diminish the chances of winning if an ESP propagates a block slowly to other miners in the broadcasting procedure. In other words, it is possible that the miner that first calculates the PoW puzzle and packages a block, may fail to get reward because someone else takes the lead in broadcasting the packaged block successfully and realizes the consensus protocol. This is because this block is likely to be discarded because of long propagation delay, which is called orphaning [6]. Following the existing work [7], we consider that the block mining time follows the Poisson distribution, and the orphaning probability on the propagation delay t_j caused by ESP j, denoted as $P_{\text{orphan}}\ (t_j)$, is approximated as:

$$P_{\text{orphan}}\ (t_j) = 1 - e^{-\lambda t_j}, \tag{2}$$

in which the parameter λ denotes the inter-arrival rate of the Poisson distribution. Thus, the successful probability of mining game for miner i on ESP j is expressed as follows:

$$W_i^j = \alpha_i^j\left(1 - P_{\text{orphan}}\ (t_j)\right) = \frac{\tau_i^j x_i^j}{E_{all}} e^{-\lambda t_j}, \tag{3}$$

where W_i^j denotes the probability that ESP j is the first one who solves the PoW problem (i.e., packages a block) and broadcasts this block successfully, that is, making it be the first consensual block. Note that an edge computing request may be sent to the remote CSP by the corresponding ESP due to his limited resource capacity. In such a case, the winning probability for the CSP is represented as follows:

$$W_i^{j\prime} = \frac{(1-\tau_i^j)x_i^j}{C_{all}} e^{-\lambda\bar{t}}. \tag{4}$$

Here, due to the different unit computing power between ESPs and CSP, the probability of solving the PoW problem is relative to the overall computing power of all miners on the CSP rather than that on ESPs. $\bar{t}$ is the propagation delay of the CSP, which is bigger than t_j for $j \in \mathcal{N}$. Hence, the winning probability of miner i on all service providers can be summarized as:

$$W_i = \sum_{j\in\mathcal{N}}\left(\frac{\tau_i^j x_i^j}{E_{all}} e^{-\lambda t_j} + \frac{(1-\tau_i^j)x_i^j}{C_{all}} e^{-\lambda\bar{t}}\right). \tag{5}$$

B. Two-Stage Stackelberg Game

We model the interactions between computing service providers and miners as a multi-leader multi-follower Stackelberg game with complete information. The ESPs, i.e., the leaders, act first by setting the unit price for their computation resources. The miners, i.e., the followers, then determine their optimal computing service request based on the prices and propagation delays of ESPs. Actually, in the first stage, the competition in ESPs forms as a non-cooperative subgame, where each ESP sets his unit price by considering miners' requests as well as other ESPs' prices. In the second stage, each miner, e.g., i, determines and sends his requested computing resources for all ESPs by taking the unit price and his budget B_i into consideration.

1) Miner Side Utility in Stage II: Assume that the unit prices of the computation resources are given by ESPs, each miner, e.g., i, decides his services demands under the budget constraint to maximize his utility, where the utility is defined as the expected reward minus the corresponding cost. The expected reward is computed by $R \cdot W_i$, in which R means the reward of successfully appending a block to the end of the existing blockchain, and W_i denotes the probability of the miner i winning the reward. On the other hand, the total cost of miner i is determined by the prices of ESPs, denoted as $\{p_1, \cdots, p_j, \cdots, p_n\}$, and this miner's service requests, i.e., $X_i = (x_i^1, x_i^2, \cdots, x_i^n)$. Based on this, we formulate the optimization problem of the miner i as follows:

$$\begin{aligned} &\textit{maximize} \quad U_i = R\cdot W_i - \sum_{j\in\mathcal{N}} p_j \cdot x_i^j \\ &\textit{subject to} \quad \sum_{j\in\mathcal{N}} p_j \cdot x_i^j \le B_i \\ &\qquad\qquad\quad \text{Eq. (1) – Eq. (5)} \end{aligned} \tag{6}$$

2) ESP Side Utility in Stage I: The profits of ESPs comes from the payments of miners. When a miner's request is accepted by an ESP, the miner must pay the ESP for his computing resource services. The utility of each ESP means that the received payment minus the corresponding cost. On the other hand, when a miner's request is uploaded to the

CSP through one ESP, this ESP's utility equals to the miner's payment minus the cost that this ESP provides for the CSP. We consider that the scale of the cloud platform is much larger than that of the edge platforms, so each edge platform's strategy will not cause an apparent impact on the cloud platform's revenue. As a result, the CSP will always keep a fixed unit price of the computing resources and there is no willingness to change his pricing mechanism. The utility of each ESP, e.g., j, is defined as $V_j = (p_j - c) \cdot E_j + (p_j - h) \cdot C_j$, where c is the unit electricity cost for providing service on the ESPs and h is the ESP's payment for renting unit computing resources from CSP. E_j denotes the total computation resources that the ESP j provides for the miners while C_j means the total computation resources that ESP j rents from the CSP, that is, $E_j = \sum_{i \in \mathcal{M}} \tau_i^j x_i^j$ and $C_j = \sum_{i \in \mathcal{M}} (1 - \tau_i^j) x_i^j$. According to this, each ESP's optimization goal is presented as follows:

$$\begin{array}{ll} maximize & V_j = (p_j - c) \cdot E_j + (p_j - h) \cdot C_j \\ subject\ to & \sum_{i \in \mathcal{M}} \tau_i^j x_i^j \leq K_j \end{array} \tag{7}$$

where K_j is the maximum computation resource capacity of ESP j. Here, when the sum of requests from all miners on ESP j exceeds his capacity constraints, he has to upload part of the received requests to the CSP. However, the long propagation delay may decrease the winning probability for the applied miners, which will in turn reduce the miners' requests. Thus, it is quite challenging for each ESP to set a suitable unit price for the computing resources so that he can maximize his utility.

We aim at finding the Stackelberg equilibrium where the payoff of ESPs and miners can be maximized simultaneously. We first define the Stackelberg equilibrium point as follows.

Definition 1 *Let $\mathbf{x}^*$ and $\mathbf{p}^*$ denote the optimal service demand vector of all the miners and optimal unit price vector of edge computing service, respectively. Then, the point $(\mathbf{x}^*, \mathbf{p}^*)$ is the Stackelberg equilibrium if the following two conditions are satisfied:*

$$V_j \left(\mathbf{p}_j^*, \mathbf{p}_{-j}^*, \mathbf{x}^* \right) \geq V_j \left(\mathbf{p}_j^{'}, \mathbf{p}_{-j}^*, \mathbf{x}^* \right) \tag{8}$$

$$U_i \left(\mathbf{x}_i^*, \mathbf{x}_{-i}^*, \mathbf{p}^* \right) \geq U_i \left(\mathbf{x}_i^{'}, \mathbf{x}_{-i}^*, \mathbf{p}^* \right) \tag{9}$$

Here, $\mathbf{p}_{-j}^*$ is the optimal pricing strategy of all ESPs except ESP j, and $\mathbf{x}_{-i}^*$is the best response service demand vector for all miners except miner i. In the next section, we will analyze the game equilibrium in the above model.

III. Stackelberg Game Equilibrium Analysis

In this section, we analyze the optimal service demand of miners as well as the profit maximization of ESPs under the Stackelberg game with a complete information model.

A. Stage II: Miners' Participation Equilibrium

Based on the definition of Stackelberg game equilibrium, as the pricing strategies of all ESPs are given, each miner determines his service demands for each ESP as the best response. We first introduce the definition of the best response.

Definition 2 *A request vector $\mathbf{x_i^*} \triangleq \left(x_i^{1*}, x_i^{2*} \ldots, x_i^{n*} \right)$ is the optimal response service demand vector of the miner subgame if $U_i \left(\mathbf{x}_i^*, \mathbf{x}_{-i}^*, \mathbf{p}^* \right) \geq U_i \left(\mathbf{x}_i^{'}, \mathbf{x}_{-i}^*, \mathbf{p}^* \right)$.*

Here, we make an assumption about the following values.

Assumption 1

$$\Delta t < \frac{\ln m}{\lambda},\ where\ \Delta t = \max \left\{ t_j - t_k \right\} \quad \forall i, k \in n \tag{10}$$

Theorem 1 *Under Assumption 1, the existence and uniqueness of miner participation equilibrium, i.e., the Nash equilibrium of Stage II in this Stackelberg game, can be guaranteed.*

Proof: The strategy space of each miner is a non-empty, compact subset of the Euclidean space. From Eq. (6), U_i is apparently continuous with the variable $\mathbf{x}_i$, which is the combination of request for each ESP, i.e., x_i^j. We take the first order and second order derivatives of Eq. (6) with respect to x_i^j as follows:

$$\frac{\partial U_i}{\partial x_i^j} = R \cdot \frac{\partial W_i}{\partial x_i^j} - p_j, \quad \frac{\partial^2 U_i}{\partial \left(x_i^j \right)^2} = R \cdot \frac{\partial^2 W_i}{\partial \left(x_i^j \right)^2}. \tag{11}$$

Based on Eq. (5), we can take the second derivative of W_i as Eq. (14), Eq. (15) and Eq. (16). It's obvious that $\frac{2\left(1-\tau_i^j\right)^2}{(C_{all})^3} e^{-\lambda \bar{t}} \left[\sum_{k=1}^n \left(1-\tau_i^k\right) x_i^k - C_{all} \right] \leq 0$, we just need to prove,

$$\sum\nolimits_{k=1}^n \tau_i^k x_i^k e^{-\lambda(t_k - tj)} - E_{all} \leq 0. \tag{17}$$

Eq. (17) can be transformed as

$$\sum\nolimits_{k=1}^n \left(\tau_i^k x_i^k e^{-\lambda(t_j - t_k)} - \sum\nolimits_{i=1}^m \tau_i^k x_i^k \right). \tag{18}$$

We take the expectation of Eq. (18) as follows:

$$\sum\nolimits_{k=1}^n \left(\tau_i^k x_i^k e^{-\lambda \Delta t} - m \tau_i^k x_i^k \right). \tag{19}$$

Based on Assumption 1, we have the following result, i.e., $\sum_{k=1}^n \left(\tau_i^k x_i^k e^{-\lambda \Delta t} - m \tau_i^k x_i^k \right) < 0$, then Eq.(15) < 0 can be guaranteed. In fact, Assumption 1 is easy to be satisfied because λ is between 0 and 1 and much less than 1. Thus, the miner participation sub-game is a concave game which always admits the Nash equilibrium.

By setting the first-order derivative of the miner's utility to 0, we have follows:

$$\text{If } \tau_i^j = 1, \frac{p_j}{R} = \frac{-1}{(E_{all})^2} \left(\sum_{k=1}^n \tau_i^j x_i^j e^{-\lambda t_k} - E_{all} e^{-\lambda t_j} \right), \tag{20}$$

$$x_i^j = \sqrt{\left(e^{-\lambda t_j} \cdot E_{all}^{-x_i^j} - \sum_{k \neq j} \tau_i^j x_i^j e^{-\lambda t_k} \right) \cdot \frac{R}{p_j}} - E_{all}^{-x_i^j}; \tag{21}$$

$$\text{if } \tau_i^j = 0, \quad \frac{p_j}{R} = \frac{e^{-\lambda \bar{t}}}{(C_{all})^2} \left(C_{all} - \sum\nolimits_{k=1}^n \left(1 - \tau_i^k\right) x_i^k \right), \tag{22}$$

$$x_i^j = \sqrt{e^{-\lambda \bar{t}} \cdot \frac{R}{p_j} \cdot C_{all}^{-i}} - C_{all}^{-x_i^j}, \tag{23}$$

$$\frac{\partial W_i}{\partial\left(x_i^j\right)} = -\frac{\tau_i^j}{\left(E_{all}\right)^2}\cdot\sum_{k=1}^n \tau_i^k x_i^k e^{-\lambda t_k} + \frac{\tau_i^j}{E_{all}} e^{-\lambda t_j} - \frac{\left(1-\tau_i^j\right)}{\left(C_{all}\right)^2}\cdot \sum_{k=1}^n \left(1-\tau_i^k\right) x_i^k e^{-\lambda\bar{t}} + \frac{\left(1-\tau_i^j\right)}{C_{all}} e^{-\lambda \bar{t}}. \tag{14}$$

$$\frac{\partial^2 W_i}{\partial\left(x_i^j\right)^2} = \frac{2\left(\tau_i^j\right)^2}{\left(E_{all}\right)^3}\cdot\sum_{k=1}^n \tau_i^k x_i^k e^{-\lambda t_k} - \frac{2\left(\tau_i^j\right)^2}{\left(E_{all}\right)^2} e^{-\lambda t_j} + \frac{2\left(1-\tau_i^j\right)^2}{\left(C_{all}\right)^3} e^{-\lambda\bar{t}}\cdot \sum_{k=1}^n \left(1-\tau_i^k\right) x_i^k - \frac{2\left(1-\tau_i^j\right)^2}{\left(C_{all}\right)^2} e^{-\lambda \bar{t}} \tag{15}$$

$$= \frac{2\left(\tau_i^j\right)^2}{\left(E_{all}\right)^3} e^{-\lambda t_j}\left[\sum_{k=1}^n \tau_i^k x_i^k e^{-\lambda(t_k - tj)} - E_{all}\right] + \frac{2\left(1-\tau_i^j\right)^2}{\left(C_{all}\right)^3} e^{-\lambda\bar{t}}\left[\sum_{k=1}^n \left(1-\tau_i^k\right) x_i^k - C_{all}\right]. \tag{16}$$

where $E_{all}^{-x_i^j} = E_{all} - x_i^j$ and $C_{all}^{-x_i^j} = C_{all} - x_i^j$. If the demands of all miners for ESP j is less than K_j, ESP j will accept the x_i^j in order to maximize its utility, i.e., $\tau_i^j = 1$. Therefore, for a rational and selfish miner, it will predict the strategies of other opponents. If $x_i^j + \sum_{k\neq i} x_k^j \leq K_j$, miner i will determines its demand for ESP j as Eq. (21). Then, we obtain the best response of miner i as Eq. (25). Since the model in this paper is relatively complex and involves multi-dimensional parameters, we cannot obtain the best response results which do not involve other users' strategy parameters for the time being. In future work, we will explore the best response results without others' strategies. However, the obtained intermediate result about the miners' best strategy will not affect the proof of the Nash equilibrium in stage I. ■

B. Stage I: Optimal Pricing Mechanism

Based on the Nash equilibrium of the computing service demand in the ESPs' subgame in Stage II, the leader of the Stackelberg game, i.e., the ESP, can optimize its pricing strategy in Stage I to maximize its profit defined in Eq. (7).

Theorem 2 *Nash equilibrium of ESPs' subgame problem exists under the condition of $p_j < 3c$ and $p_j < 3h$.*

Proof: By taking the first order and second order derivatives of Eq. (7), we have Eq. (26) and Eq. (27).

Specifically, when ESP j accepts the demand x_i^j, i.e., $\tau_i^j = 1$, for easy of illustration, we denote A as $\sqrt{\left(e^{-\lambda t_k \cdot E_{all}^{-x_i^j}} - \sum_{k\neq j} \tau_i^j x_i^j e^{-\lambda t_k}\right)\cdot R}$, which is nothing to do with p_j, so we can obtain that:

$$\mathbf{x_i^j}^* = \frac{A}{\sqrt{p_j}} - E_{all}^{-x_i^j}; \tag{28}$$

Similarly, we denote B as $\sqrt{e^{-\lambda\bar{t}}\cdot R\cdot C_{\text{all}}^{-i}}$ in the case where ESPs upload demand x_i^j to the CSP, i.e., $\tau_i^j = 0$:

$$\mathbf{x_i^j}^* = \frac{B}{\sqrt{p_j}} - C_{all}^{-x_i^j}; \tag{29}$$

By substituting Eq. (28) and Eq. (29) into Eq. (27), we can gain Eq. (30) .

Therefore, we can conclude that when the condition $p_j < 3c$ and $p_j < 3h$ are satisfied, the negativity of $\frac{\partial^2 V_j}{\partial (p_j)^2}$ can be guaranteed. In other words, ESP side subgame problem is convex with respect to p_j under this condition. The solution's uniqueness further guarantees the global convergence and SE is achieved given that the Nash equilibrium is found in the leader stage. Hence, the theorem holds. ■

Algorithm 1 Asynchronous Best Response

Input: Any feasible price $P = \{p_1, p_2, \ldots, p_n\}$, miners' demands $X = \{X_1, X_2, \ldots, X_m\}$, and the threshold ϵ

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for iteration k do
    storing last iteration P^(k-1);
    for each miner i do
        receiving the pricing strategy P = {p_1, p_2, ..., p_n};
        predicting the optimal requests of other miners;
        calculating x_i^j(k) = x_i^j(k-1) + Δ ∂U_i(X_{-i}^(k-1), X_i^(k-1), P) / ∂x_i^j;
        deciding his request x_i^(k) = {x_i^1, x_i^2, ..., x_i^n};
    for each ESP j do
        update the price with a step δ;
        predicting miners' optimal requests x* for each ESP;
        if the new price brings more profit then
            p_j^(k) ← p_j^(k) + δ;
    if ||P^(k) − P^(k-1)|| < ϵ then
        return P^(k) and x*;
    else
        k ← k + 1;
```

We take advantage of a classic distributed algorithm called Asynchronous Best Response to find the Nash equilibrium point in ESPs' subgame, where ESP is engaged in a gradient ascent process to maximize its utility. We use the gradient addition method to adjust the price strategies of ESPs and the miners' demands policy in each round. These operations are conducted in each round of iteration until the difference of the Frobenius norms of the price strategy in previous round and that in this round is less than a given threshold. The point now is the Nash equilibrium point that we are looking for. Finally, Algorithm 1 terminates and outputs the results.

IV. Performance Evaluation

We first simulate the multi-leader multi-follower Stackelberg game between miners and ESPs and further verify the practicality of our proposed utility function of miners. Then, numerical examples to certificates the system model we proved

$$\mathbf{x_i^{j*}} = \begin{cases} \sqrt{\left(e^{-\lambda t_k \cdot E_{all}^{-x_i^j}} - \sum_{k \neq j} \tau_i^j x_i^j e^{-\lambda t_k}\right) \cdot \frac{R}{p_j}} - E_{\text{all}}^{-x_i^j}, \text{ if } x_i^j + \sum_{k \neq i} x_k^j \leq B_j \\ \sqrt{e^{-\lambda \bar{t}} \cdot \frac{R}{p_j} \cdot C_{\text{all}}^{-i}} - C_{\text{all}}^{-x_i^j}, \text{ otherwise} \end{cases} \tag{25}$$

$$\frac{\partial V_j}{\partial p_j} = E_j + (p_j - c) \sum\nolimits_{i \in M} \tau_i^j \frac{\partial x_i^j}{\partial p_j} + C_j + (p_j - h) \sum\nolimits_{i \in M} \left(1 - \tau_i^j\right) \frac{\partial x_i^j}{\partial p_j}. \tag{26}$$

$$\frac{\partial^2 V_j}{\partial \left(p_j\right)^2} = \sum_{i \in M} \tau_i^j \frac{\partial x_i^j}{\partial p_j} + (p_j - c) \cdot \sum_{i \in M} \tau_i^j \frac{\partial^2 x_i^j}{\partial \left(p_j\right)^2} + \sum_{i \in M} \left(1 - \tau_i^j\right) \frac{\partial x_i^j}{\partial p_j} + (p_j - h) \cdot \sum_{i \in M} \left(1 - \tau_i^j\right) \frac{\partial^2 x_i^j}{\partial \left(p_j\right)^2}. \tag{27}$$

$$\frac{\partial^2 V_j}{\partial \left(p_j\right)^2} = \sum_{i \in M} \left(\tau_i^j \left[\frac{(p_j - c) \cdot \frac{3A}{4} - \frac{A}{2} \cdot p_j}{\left(p_j\right)^{5/2}} \right] + \left(1 - \tau_i^j\right) \left[\frac{(p_j - h) \cdot \frac{3B}{4} - \frac{B}{2} \cdot p_j}{\left(p_j\right)^{5/2}} \right] \right) \tag{30}$$

is reasonable. We assume that the parameter of propagation delay λ is fixed as $\frac{1}{600}$, as introduced in the work [8]. In addition, when mentioning the prices set by the ESPs, no matter whether they are optimized or not, $p_j > c$, $p_j > h$ and $\bar{t} >> t_j$ always hold.

A. Influence at ESP side

We address the comparison of how capacity affect utility of ESP in different numbers of miners or different numbers of ESPs cases. As illustrated in Fig. 2, we observe that the profit of an ESP rises with the increase of its own capacity. This is because more capacity allows the ESP to accept more requests and reduces the amount of the computing tasks that are uploaded to the CSP. Here, the cost for uploading requests to the CSP is bigger than that of running the tasks on ESP locally. Thus, more tasks implemented on ESP enhances the profit of ESP. We also see that the utility of ESP increases as the number of miners increases but drops with the number of ESPs conversely. This is due to the fact that a greater number of miners will cause more computing service demands and the competition will be more intense in the case including more miners. In such settings, the miners are willing to request more computing resources to upgrade their winning probability. However, with the fixed number of miners, the budget of miners is limited, which means the total requests are limited under their budget constraint. Thus, more ESPs share the limited benefits, resulting in a decrease in the average profit of each ESP. Fig. 3 shows the ESPs' utility rises when the average budget of miners B_i varies from 60 to 120. This is because the miners have more money to purchase the computing service, which further increases the utility of ESPs.

B. Influence at Miner Side

We also study the miners' utility with the change of miners' budgets and ESPs' capacity. Fig. 4 indicates that, when the budgets of other miners are fixed as a certain value, the increase of one miner's budget will result the higher utility of this miner. Obviously, increasing a miner's budget can enable him to purchase more computing resources and increase the probability of solving the PoW problem, which further enhances his profits. We find that as the number of ESPs is fixed, the miner's utility decreases with the increase of number of miners. This is because a more competitive game among more miners will lead to the lower probability of each competitor, and then decrease the utility of each miner. Fig. 4 demonstrates that if the number of ESPs increases, each miner will get more profits. The reason is that more ESPs bring higher total capacity which will allow more demands to be accepted by ESPs, resulting in the higher probability of winning the game due to the shorter propagation delay of ESPs than CSPs. Nevertheless, when the certain miner's budget is set as 90, changing the number of ESPs from 4 to 5 has no effect on the growth of miner's benefit. This is due to the fact that, when the miner's budget is under a certain amount, simply increasing the number of ESPs has no help for a miner to purchase more computing service owing to the limited budget. As a result, the utility of one miner holds constantly until the budget grows up to a higher value.

Fig. 5 shows the impact of ESPs' capacity on miners' utility. The average of miners' utility increases as the average capacity of ESPs increases at first. While the average capacity of ESPs reaches a certain value, the changes in miners' utility tends to be flat. Miners specify their own demand strategy based on their own budget and the prediction of other competitors' anticipation. More capacity of ESPs facilitate the acceptance of miners' requests. Therefore, within a limited budget, increasing ESPs' capacity will bring a certain benefit growth. However, due to the limited budget of miners, they cannot pay for too much computing services, so the continuous growth of ESPs' capacity brings no benefit in their utility when their requests have reached the upper limit of their budgets.

C. System Performance Evaluation

We then evaluate the system performance for miners and providers under the proposed algorithm, as illustrated in Fig. 6, Fig. 7, and Fig. 8. We evaluate the utility of miners and ESPs as well as the computing service demanded by miners with the changes of the reward parameters. With the increase of the reward, all values of total demanded computing service, ESPs' utility and miners' utility grow up obviously. Moreover, we can find that the propagation delay shows a positive correlation

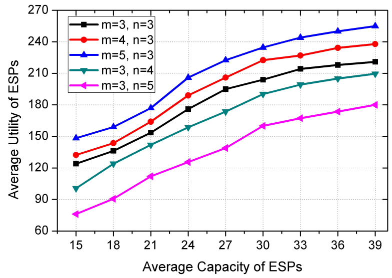

Fig. 2. ESP' Utility vs. Capacity.

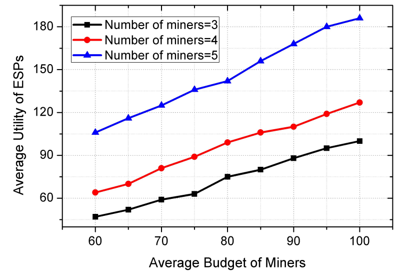

Fig. 3. ESP' Utility vs. Miners' Budget.

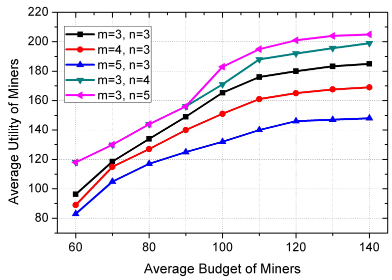

Fig. 4. Miners' Utility vs. Budget.

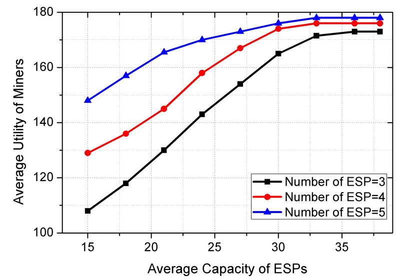

Fig. 5. Miners' Utility vs. ESP' Capacity.

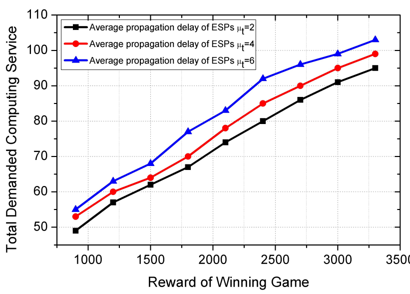

Fig. 6. Total Demands vs. Reward.

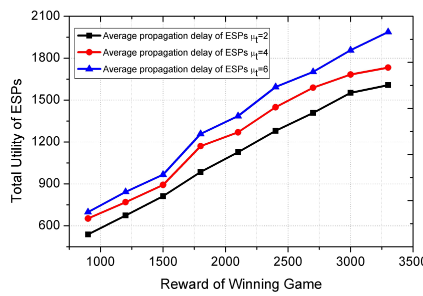

Fig. 7. ESP' Utility vs. Reward.

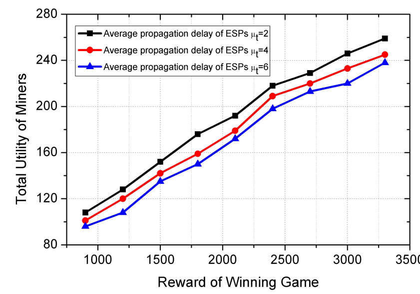

Fig. 8. Miners' Utility vs. Reward.

trend with the computing service demands and utility of ESPs in Fig. 7 and Fig. 8. This can be explained in the Eq. (5), i.e., the probability of a miner winning the mining game W_i consists of two part. One factor is the probability of the block he packages becoming the first consensus, which is negatively related to the propagation delay. The increase of the average propagation delay causes the decrease of W_i. Thus, the miner has to demand more computing service to compete for others. This will in turn increase the profits of the ESPs. However, demanding for more computing service makes the miners pay more, which will reduce the utility. This is why the utility of miners decreases with the increase of ESPs' propagation delay. Fig. 9 illustrates the relation between the precision threshold ϵ versus the number of iteration rounds for convergence when the numbers of miners vary. From Fig. 9, we see that the iteration rounds are exponentially related to the threshold ϵ, and with the increase of the number of miners, the iteration rounds for convergence grow up as well.

V. Related Work

A. Mining Game in Blockchain Network

Game theory is widely used to solve many network problems like [9–12]. A large number of studies have been developed in mining schemes management for blockchain networks [13, 14]. In [15], the authors designed a noncooperative game among the miners. The miner's strategy is to choose the number of transactions to be included in a block, where solving the PoW puzzle for mining is modeled as a Poisson process. Then, [16] modeled the mining process as a sequential game where the miners compete for mining reward sequentially among them. Similarly, the authors in [17] formulated the stochastic game for modeling the mining process, where miners decide on which blocks to extend and whether to propagate the mined block.

B. Cloud/Edge Computing Based Blockchain

Cloud providers offer virtually unlimited computation and storage resources on demand, allowing for the elasticity and scalability of applications deployed. While the mobile edge computing, which is emerging as an effective way to mitigate the problem of long latency and the current network architecture, has attracted more attention. For example, [18] proposed a multi-leader multi-follower Stackelberg game to formulate the two-tier offloading problem. With the development of cloud computing and edge computing, miners prefer offloading the PoW computations to local edge service due to the limited computing resource on their mobile terminals [19–22]. In [23], the authors considered a blockchain-based mining game model with an ESP and a CSP in two situations, i.e., the ESP

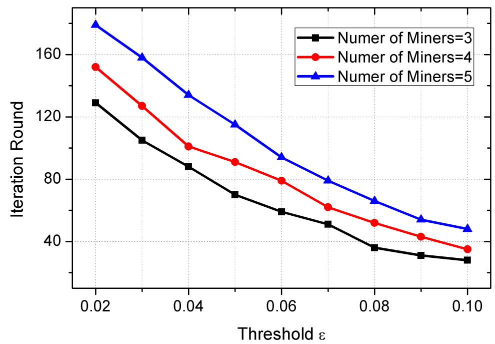

Fig. 9. Total Iteration Rounds vs. Threshold.

is connected (to the CSP) or standalone and then analyzed the Stackelberg equilibrium in these model. While in real scenarios, it is common for multiple ESPs to compete for pricing and sell resources which is not considered in this work. All previous studies did not take into account the different propagation delays due to the geographic locations of ESPs which led to the different probability of miners winning. Therefore, this motivates us to take a step further to reconsider the resource management in mobile environment.

VI. Conclusion

In this paper, we investigate the resource pricing and scheduling problem in the edge-assisted blockchain mining networks by using the multi-leader multi-follower Stackelberg game theory. In particular, we first propose the edge computing model where ESPs has different propagation delays according to his geographical location. Then, we analyze the utility of both miners and ESPs in such a model, and further discuss the existence and the uniqueness of Stackelberg Equilibrium (SE). Furthermore, we propose an algorithm to achieve the SE and conduct extensive simulations to validate the convergence as well as evaluate the network performance. In the future work, we will further explore the the best response results without others' strategies and also consider the CSP's utility in the Stackelberg model. Moreover, the propagation delay caused by the size of the block as well as the geographic location factors will be comprehensively considered in the next work.

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References

[1] "Crypto-currency market capitalizations," https://coinmarketcap.com 2017.

[2] X. Han, Y. Yuan, and F. Wang, "A fair blockchain based on proof of credit," *IEEE Transactions on Computational Social Systems*, vol. 6, no. 5, pp. 922–931, 2019.

[3] N. Bozic, G. Pujolle, and S. Secci, "A tutorial on blockchain and applications to secure network control-planes," in *3rd Smart Cloud Networks Systems*, 2016.

[4] G. Gao, M. Xiao, J. Wu, H. Huang, S. Wang, and G. Chen, "Auction-based vm allocation for deadline-sensitive tasks in distributed edge cloud," *IEEE Transactions on Services Computing*, pp. 1–1, 2019.

[5] L. Wan, D. Eyers, and H. Zhang, "Evaluating the impact of network latency on the safety of blockchain transactions," in *IEEE International Conference on Blockchain*, 2019.

[6] Z. Xiong, J. Kang, D. Niyato, P. Wang, and H. V. Poor, "Cloud/edge computing service management in blockchain networks: Multi-leader multi-follower game-based admm for pricing," *IEEE Transactions on Services Computing*, vol. 13, no. 2, pp. 356–367, 2020.

[7] Z. Xiong, Y. Zhang, D. Niyato, P. Wang, and Z. Han, "When mobile blockchain meets edge computing," *IEEE Communications Magazine*, vol. 56, no. 8, pp. 33–39, 2018.

[8] H. Nicolas, "The bitcoin mining game," *Ssrn Electronic Journal*, 2014.

[9] H. Huang, Y.-e. Sun, K. Xing, H. Xu, X. Xu, and L. Huang, "Truthful multi-unit double auction for spectrum allocation in wireless communications," in *WASA*. Springer, 2012.

[10] G. Gao, M. Xiao, J. Wu, L. Huang, and C. Hu, "Truthful incentive mechanism for nondeterministic crowdsensing with vehicles," *IEEE Transactions on Mobile Computing*, vol. 17, no. 12, pp. 2982–2997, 2018.

[11] J. Myung, Y. Chen, K. J. R. Liu, and J. Kang, "Non-cooperative feedback control game for secondary transmitter in cognitive radio network," *IEEE Signal Processing Letters*, vol. 20, no. 6, pp. 571–574, 2013.

[12] X. Kang and Y. Wu, "Incentive mechanism design for heterogeneous peer-to-peer networks: A stackelberg game approach," *IEEE Transactions on Mobile Computing*, vol. 14, no. 5, pp. 1018–1030, 2015.

[13] P. De Giovanni, "Blockchain and smart contracts in supply chain management: A game theoretic model," *International Journal of Production Economics*, vol. 228, p. 107855, 2020.

[14] E. Altman, D. Menasché, A. Reiffers-Masson, M. Datar, S. Dhamal, C. Touati, and R. El-Azouzi, "Blockchain competition between miners: a game theoretic perspective," *Frontiers in Blockchain*, vol. 2, p. 26, 2020.

[15] N. Houy, "The bitcoin mining game," *Available at SSRN 2407834*, 2014.

[16] J. Beccuti and C. Jaag, "The bitcoin mining game: On the optimality of honesty in proof-of-work consensus mechanism," *Swiss Economics Working Paper 0060*, 2017.

[17] A. Kiayias, E. Koutsoupias, M. Kyropoulou, and Y. Tselekounis, "Blockchain mining games," in *ACM Conference on Economics and Computation*, 2016.

[18] J. Liu, L. Li, F. Yang, X. Liu, X. Li, X. Tang, and Z. Han, "Minimization of offloading delay for two-tier uav with mobile edge computing," in *Wiley. WCMC*, 2019.

[19] D. C. Nguyen, P. N. Pathirana, M. Ding, and A. Seneviratne, "Privacy-preserved task offloading in mobile blockchain with deep reinforcement learning," *IEEE Transactions on Network and Service Management*, vol. 17, no. 4, pp. 2536–2549, 2020.

[20] S. Guo, Y. Dai, S. Guo, X. Qiu, and F. Qi, "Blockchain meets edge computing: Stackelberg game and double auction based task offloading for mobile blockchain," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 5, pp. 5549–5561, 2020.

[21] M. Liu, F. R. Yu, Y. Teng, V. C. M. Leung, and M. Song, "Joint computation offloading and content caching for wireless blockchain networks," in *IEEE INFOCOM Workshops*, 2018.

[22] M. Xiao, J. Wu, H. Huang, L. Huang, and C. Hu, "Deadline-sensitive user recruitment for probabilistically collaborative mobile crowdsensing," in *IEEE ICDCS*, 2016.

[23] S. Jiang, X. Li, and J. Wu, "Multi-leader multi-follower stackelberg game in mobile blockchain mining," *IEEE Transactions on Mobile Computing*, pp. 1–1, 2020.